An Optimal Real-Time Voltage and Frequency Scaling for Uniform Multiprocessors

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Motivation

Chip multiprocessors are the way to deal with increasing computational load in embedded real-time systems

- Power consumption, heat dissipation, and other physical constraints render single processors impractical

Power consumption is a concern in battery-powered real-time systems

- battery life time
- battery weight
Overview

Voltage and frequency scaling (VFS) allows reducing the power consumption of a processor, and its speed.

VFS in real-time systems must ensure that the system remains schedulable.

Existing VFS algorithms for multiprocessors leave unused idle capacity.

- processor constraints
- algorithm constraints

Growing Minimum Frequency (GMF) algorithm achieves better power efficiency.

- removes algorithm constraint
- reduces impact of processor constraints
Problem Description

Given:
- multiprocessor platform supporting independent VFS, and
- a set of periodic tasks with implicit deadlines

Compute:
- frequency assignment that minimizes power consumption while meeting tasks’ deadlines
Processor Power-Frequency Relationship

Dynamic power in processors is proportional to the product of the processor frequency and the square of the supply voltage.

\[ P \propto V^2 f \]

In VFS, power can be reduced by reducing the frequency, which allows a corresponding reduction in the voltage.

Since voltage is proportional to the frequency we can approximate as

\[ P \propto f^3 \]
Task and Platform Model

Tasks

\( n \): number of tasks
\( C_i \): execution time of task \( \tau_i \), measured at the highest frequency
\( T_i \): period of task \( \tau_i \)
\( D_i = T_i \): implicit deadlines
\( u_i = \frac{c_i}{T_i} \): utilization of task \( \tau_i \)
\( U = \sum_{i=1}^{n} u_i \): total utilization

Platform:

\( m \): number of processors (all identical)
\( f_i \): normalized frequency (1 = highest frequency) for processor \( i \)
Uniform Frequency Scaling

- All processors assigned the same frequency
- Tasks scheduled with an optimal global scheduler (e.g., LNREF)

\[ f = \max\left(\frac{U}{m}, u_1, \ldots, u_n\right) \]

Heavy tasks may constrain frequency scaling

Idle capacity
Non-Uniform Frequency Scaling

- Processor frequencies are assigned independently

Decide Independent Frequency algorithm avoids heavy task bottleneck [Funaoka 2008]

- Task $\tau_i$ is heavy if its utilization would drive up the uniform frequency assignment for the remaining processors, i.e. $u_i > \frac{\sum_{j=i}^{n} u_j}{m-i+1}$
- Each heavy task is assigned its own processor
- Remaining light tasks globally scheduled in remaining processors with uniform frequency assignment
Non-Uniform Frequency Scaling

Decide Independent Frequency is optimal if frequency can be scaled continuously (i.e. to any frequency in a range)

- \( f_1 = 1 \)
- \( f_2 = 1 \)
- \( f_3 = 1 \)
- \( f_4 = 1 \)
- \( f_5 = 1 \)

- \( \tau_1: 100\% @ 1 \)
- \( \tau_2: 90\% @ 1 \)
- \( \tau_3: 60\% @ 1 \)
- \( \tau_4: 50\% @ 1 \)
- \( \tau_5: 10\% @ 1 \)

VFS processor partitions

uniform frequency scaling for remaining tasks and processors
Discrete Frequency Steps

However, processors support a limited number of frequencies.

- DIF is not optimal in that setting
- Computing the optimal partition of processors and the frequency assignment is NP-Hard

*Example:* supported frequencies: 1, .75, .5

```
<table>
<thead>
<tr>
<th>Frequency</th>
<th>Task 1</th>
<th>Task 2</th>
<th>Task 3</th>
<th>Task 4</th>
<th>Task 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1=1</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f2=1</td>
<td>90%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f3=.75</td>
<td>80%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f4=.75</td>
<td>67%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f5=.75</td>
<td>13%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```
Achieving Better Power Efficiency

Two problems

- Discrete frequency steps force leaving idle capacity in processor partitions
- Unused capacity in a processor partition cannot be used by tasks assigned to other partitions

Observation: if we can optimally schedule tasks allowing them to migrate between processors running at different frequencies we can do better

- avoid the set partition problem (and its computational complexity)
- achieve better power efficiency
  - no fragmentation of platform capacity
  - capacity left by heavy tasks is not wasted
U-LLREF

U-LLREF [Funk 2010] is an optimal global scheduling algorithm for uniform multiprocessors

- an extension of LLREF (a DP-fair algorithm)
- processors can run at different frequencies

A task set is schedulable by U-LLREF on a platform if the following holds

\[
\sum_{i=1}^{k} u_i \leq \sum_{i=1}^{k} f_i \quad \forall k \in \{1, \ldots, m - 1\}
\]

\[
\sum_{i=1}^{n} u_i \leq \sum_{i=1}^{m} f_i
\]

where \( u_1 \geq \cdots \geq u_n \) and \( f_1 \geq \cdots \geq f_m \)
Growing Minimum Frequency Algorithm

Overview: satisfy each condition of the U-LLREF test using the most power efficient assignment of frequencies (lowest possible and distributed as uniformly as possible)

assign the lowest frequency to all the processors

for \( k = 1 \) to \( m \) do

    while \( k^{th} \) U-LLREF condition not satisfied do

        increase the frequency of the slowest processors in subset 1..\( k \) to the next frequency step

    end while

end for
GMF Example 1

\( k = 1 \)

\( \tau_1 \)  \( \tau_2 \)  \( \tau_3 \)  \( \tau_4 \)  \( \tau_5 \)

0.5  0.5  0.5  0.5

\[ \sum_{i=1}^{k} u_i = 1.0 > \sum_{i=1}^{k} f_i = 0.5 \]
GMF Example 2

$k = 1$

slowest

\[
\begin{align*}
0.75 & \quad 0.5 & \quad 0.5 & \quad 0.5 \\
\end{align*}
\]

\[
\begin{align*}
\tau_1 & \quad \tau_2 & \quad \tau_3 & \quad \tau_4 & \quad \tau_5 \\
1.0 & \quad 0.9 & \quad 0.6 & \quad 0.5 & \quad 0.1 \\
\end{align*}
\]

\[
\sum_{i=1}^{k} u_i = 1.0 > \sum_{i=1}^{k} f_i = 0.75
\]
$$k = 1$$

<table>
<thead>
<tr>
<th>$\tau_1$</th>
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$$\sum_{i=1}^{k} u_i = 1.0 = \sum_{i=1}^{k} f_i = 1.0$$
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GMF Example 4

\[ k = 2 \]

\[ \sum_{i=1}^{k} u_i = 1.9 \quad > \quad \sum_{i=1}^{k} f_i = 1.5 \]
$$k = 2$$

$$\sum_{i=1}^{k} u_i = 1.9 > \sum_{i=1}^{k} f_i = 1.75$$
GMF Example 6

\( k = 2 \)

\[
\sum_{i=1}^{k} u_i = 1.9 < \sum_{i=1}^{k} f_i = 2.0
\]
GMF Example 6

$k = 3$

\[
\sum_{i=1}^{k} u_i = 2.5 \quad = \quad \sum_{i=1}^{k} f_i = 2.5
\]
GMF Example 7

\[ k = 4 = m \]

The diagram shows a set of processors with their respective frequencies:

- \( \tau_1: 1.0 \)
- \( \tau_2: 1.0 \)
- \( \tau_3: 0.5 \)
- \( \tau_4: 0.5 \)

The task set consists of five tasks with different frequencies:

- \( f_1 \in \{0.5, 0.75, 1\} \)

The equation for the GMF is:

\[
\sum_{i=1}^{n} u_i = 3.1 > \sum_{i=1}^{m} f_i = 3.0
\]
GMF Example 8

\[ k = 4 = m \]

\[ \sum_{i=1}^{n} u_i = 3.1 \ < \ \sum_{i=1}^{m} f_i = 3.25 \]
GMF Example 9

In this case, frequency assignment is the same as in the Exhaustive partition search.
Evaluation

Randomly generated 15,000 tasksets
- utilization level ranging from 0.5 to 4 in steps of 0.25
- 1,000 tasksets for each utilization level
- each taskset composed of tasks with random uniform utilization

Used frequencies and voltages of three different quad-core processors

Computed frequency assignment and corresponding power with different multiprocessor VFS algorithms
- Decide Independent Frequency [Funaoka 2008]
- Exhaustive partition/frequency assignment search
- GMF
- Optimal (exhaustive frequency assignment w/o partitions)
Evaluation Results

- DIF
- Exhaustive
- GMF
- Optimal

Utilization vs. Power

- Power range: 0.2 to 1.0
- Utilization range: 0.5 to 4.0

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Optimality

GMF is optimal when the supported frequency steps are uniform.

Proof intuition:
- Any frequency step we choose is the same in terms of speed
- Increasing the frequency of the slowest processor requires the smallest power increase
- The optimal frequency assignment for the first $i$ conditions bounds from below the optimal assignment for the $i+1$ conditions
- GMF assigns frequencies as even as possible within that bound
Optimality

With non-uniform frequency steps, GMF may not optimal

- When the power steps associated frequency steps are non-decreasing

We have observed that for some platforms with non-uniform frequency steps GMF is still optimal

if frequency increase $\delta_2$ is enough to satisfy scheduling condition, taking slowest increase $\delta_1$ is not optimal
Conclusion

Growing Minimum Frequency (GMF) algorithm computes the optimal frequency assignment to minimize the power consumption of a real-time periodic taskset in a multiprocessor platform.

Evaluation results show up to 30% improvement over previous algorithms.

Avoiding partitioning allows GMF to achieve better power efficiency than optimal partitioned approaches.